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Mathematical behavior of the solutions of a class of hyperbolic-type equation

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Abstract

In this paper, we consider hyperbolic-type equations with initial and Dirichlet boundary conditions in a bounded domain. Under some suitable assumptions on the initial data and source term, we obtain nonexistence of global solutions for arbitrary initial energy.

Keywords: *Hyperbolic equation, nonexistence, damping term.*

Hiperbolik tipten bir denklemin çözümlerinin matematiksel davranışı

Özet

Bu makalede sınırlı bir bölgede hiperbolik tipten başlangıç ve Dirichlet sınır koşullu problem ele alınmıştır. Başlangıç ve kaynak terim üzerine bırakılan bazı uygun koşullar altında çözümlerin global yokluğu keyfi başlangıç enerjisi için çalışılmıştır.

Anahtar kelimeler: *Hiperbolik denklem, yokluk, damping terim.*

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1. Introduction

Let Ω be a bounded domain with smooth boundary $\partial\Omega$ in R^n. We study the following hyperbolic equation

$$\begin{cases} u_{tt} + \Delta^2 u - \Delta u + u_t = |u|^{q-1}u, & (x,t) \in \Omega \times (0,T), \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), & x \in \Omega \\ u(x,t) = \frac{\partial}{\partial \vartheta} u(x,t) = 0, & x \in \partial\Omega \end{cases} \tag{1}$$

where $q \geq 1$ is real numbers, ϑ is the outer normal.

When without fourth order term $\Delta^2 u$, the equation (1) reduces to the following form

$$u_{tt} - \Delta u + u_t = |u|^{q-1}u. \tag{2}$$

Many authors has been extensively studied existence and blow up result (see[1-5]). The first serious study on the equation (2) was made by Levine [2,3]. He introduced the concavity method and studied that nonexistence of global solutions with negative initial energy. Later, Georgiev and Todorova [1] extended Levine's result. In 1999, Vitillaro [5] improved the result of Georgiev and Todorova for positive initial energy.

Without the $-\Delta u$ term, the equation (1) can be written in the following form

$$u_{tt} + \Delta u^2 + u_t = |u|^{q-1}u. \tag{3}$$

Messaoudi [6] studied the local existence and blow up of the solution to the equation (3). Wu and Tsai [7] obtained global existence and blow up of the solution of the problem (3). Later, Chen and Zhou [8] studied blow up of the solution of the problem (3) for positive initial energy.

In this paper, we prove the nonexistence of global solutions for the problem (1). There are several books (e.g. [9-11]) with very detailed and extensive study on blow up theory.

This paper is organized as follows. In Section 2, we present some lemmas and notations needed later of this paper. In Section 3 and 4, nonexistence of the solution is discussed.

2. Preliminaries

In this section, we will give some lemmas and a local existence theorem. Let $\|.\|$ and $\|.\|_p$ denote the usual $L^2(\Omega)$ norm and $L^p(\Omega)$ norm, respectively. Also, $W_0^{m,2}(\Omega) = H_0^m(\Omega)$ is a Hilbert spaces (see [12, 13], for details)

Lemma 1 (Sobolev-Poincare inequality) [12]. Let p be a number with $2 \leq p < \infty$ $(n = 1,\ 2)$ or $2 \leq p \leq \frac{2n}{n-2}$ $(n \geq 3)$, then there is a constant $C_* = C_*(\Omega, p)$ such that

$\|u\|_p \leq C_* \|\nabla u\|$ for $u \in H_0^1(\Omega)$.

We define the energy function as follows

$$E(t) = \frac{1}{2}\|u_t\|^2 + \frac{1}{2}(\|\nabla u\|^2 + \|\Delta u\|^2) - \frac{1}{q+1}\|u\|_{q+1}^{q+1}. \tag{4}$$

Lemma 2. $E(t)$ is a nonincreasing function for $t \geq 0$ and

$$E'(t) = -\|u_t\|^2 \leq 0. \tag{5}$$

Proof. Multiplying the equation of (1) by u_t and integrating over Ω using integrating by parts, we get

$$E(t) - E(0) = -\int_0^t \|u_\tau\|^2 \, d\tau \quad \text{for } t \geq 0. \tag{6}$$

Next, we state the local existence theorem of problem (1), whose proof can be found in [14].

Teorem 3 (Local existence). Suppose that $(u_0, u_1) \in H_0^2(\Omega) \times L^2(\Omega)$ holds, then there exists a unique solution u of (1) satisfying

$$u \in C\big([0,T); H_0^2(\Omega)\big), \qquad u_t \in C\big([0,T); L^2(\Omega)\big) \cap L^{p+1}\big(\Omega \times (0,T)\big).$$

Moreover, at least one of the following statements holds:

(i) $T = \infty$,
(ii) $\|u_t\|^2 + \|\Delta u\|^2 \to \infty$ as $t \to T^-$.

3. Nonexistence of solutions with arbitrary initial energy

In this section, we prove nonexistence of the solution for the problem (1) with negative and nonnegative initial energy.

Lemma 4 [15]. Let us have $\delta > 0$ and let $B(t) \in C^2(0,\infty)$ be a nonnegative function satisfying

$$B''(t) - 4(\delta+1)B'(t) + 4(\delta+1)B(t) \geq 0. \tag{7}$$

If

$$B'(0) > r_2 B(0) + K_0, \tag{8}$$

with $r_2 = 2(\delta+1) - 2\sqrt{(\delta+1)\delta}$, then $B'(t) > K_0$ for $t > 0$, where K_0 is a constant.

Lemma 5 [15]. If $H(t)$ is a nonincreasing function on $(t_0, \infty]$ and satisfies the differential inequality

$$[H'(t)]^2 \geq a + b[H(t)]^{2+\frac{1}{\delta}}, \text{ for } t \geq t_0, \tag{9}$$

where $a > 0, b \in R$, then there exists a finite time T^* such that

$$\lim_{t \to T^{*-}} H(t) = 0.$$

Upper bounds for T^* are estimated as follows:

(i) If $b < 0$ and $H(t_0) < min\left\{1, \sqrt{-\frac{a}{b}}\right\}$ then

$$T^* \leq t_0 + \frac{1}{\sqrt{-b}} \ln \frac{\sqrt{-\frac{a}{b}}}{\sqrt{-\frac{a}{b}} - H(t_0)}.$$

(ii) If $b = 0$, then

$$T^* \leq t_0 + \frac{H(t_0)}{H'(t_0)}.$$

(iii) If $b > 0$, then

$$T^* \leq \frac{H(t_0)}{\sqrt{a}} \text{ or } T^* \leq t_0 + 2^{\frac{3\delta+1}{2\delta}} \frac{\delta c}{\sqrt{a}} \left[1 - \left(1 + cH(t_0)\right)^{-\frac{1}{2\delta}}\right]$$

where $c = \left(\frac{a}{b}\right)^{2+\frac{1}{\delta}}$.

Definition 6. A solution u of (1) is called blow up if there exists a finite time T^* such that

$$\lim_{t \to T^{*-}} \left[\int_\Omega u^2 \, dx + \int_0^t \int_\Omega u^2 \, dx d\tau\right] = \infty. \tag{10}$$

Let

$$a(t) = \int_\Omega u^2 \, dx + \int_0^t \int_\Omega u^2 \, dx d\tau, \text{ for } t \geq 0. \tag{11}$$

Lemma 7. Assume $\frac{q-1}{4} \geq \delta \geq 0$, then we have

$$a''(t) \geq 4(\delta + 1) \int_\Omega u_t^2 \, dx - 4(2\delta + 1)E(0) + 4(2\delta + 1) \int_0^t \|u_\tau\|^2 \, d\tau. \tag{12}$$

Proof. By differentiating (11) with respect to t, we have

$$a'(t) = 2\int_{\Omega} uu_t\,dx + \|u\|^2, \tag{13}$$

$$\begin{aligned} a''(t) &= 2\int_{\Omega} u_t^2\,dx + 2\int_{\Omega} u\,u_{tt}\,dx + 2\int_{\Omega} u\,u_t\,dx \\ &= 2\left(\|u_t\|^2 + \|u\|_{q+1}^{q+1}\right) - 2(\|\nabla u\|^2 + \|\Delta u\|^2). \end{aligned} \tag{14}$$

Then from (1) and (14) we have

$$\begin{aligned} a''(t) = {} & 4(\delta+1)\int_{\Omega} u_t^2\,dx - 4(2\delta+1)E(0) \\ & +4\delta(\|\nabla u\|^2 + \|\Delta u\|^2) + \left(2 - \frac{4(2\delta+1)}{q+1}\right)\|u\|_{q+1}^{q+1} \\ & +4(2\delta+1)\int_0^t \|u_\tau\|_2^2\,d\tau. \end{aligned}$$

Since $\frac{q-1}{4} \geq \delta \geq 0$, we obtain (12) .

Lemma 8. Assume $\frac{q-1}{4} \geq \delta \geq 0$ and one of the following statements are satisfied

(i) $E(0) < 0$ and $\int_{\Omega} u_0 u_1\,dx > 0$,

(ii) $E(0) = 0$ and $\int_{\Omega} u_0 u_1\,dx > 0$,

(iii) $E(0) > 0$ and

$$a'(0) > r_2\left[a(0) + \frac{K_1}{4(\delta+1)}\right] + \|u_0\|^2 \tag{15}$$

holds.

Then $a'(t) > \|u_0\|^2$ for $t > t^*$, where $t_0 = t^*$ is given by (16) in case (i) and $t_0 = 0$ in cases (ii) and (iii), where K_1 and t^* are defined in (20) and (16), respectively.

Proof. (i) If $E(0) < 0$, then from (12), we have

$$a'(t) \geq 2\int_{\Omega} u_0 u_1\,dx + \|u_0\|^2 - 4(2\delta+1)E(0)t, \;\; t \geq 0.$$

Thus we get $a'(t) > \|u_0\|^2$ for $t > t^*$, where

$$t^* = max\left\{\frac{a'(0) - \|u_0\|^2}{4(2\delta+1)E(0)}, 0\right\}. \tag{16}$$

(ii) If $E(0) = 0$ and $\int_{\Omega} u_0 u_1\,dx > 0$, then $a''(t) \geq 0$ for $t \geq 0$.
We have $a'(t) > \|u_0\|^2,\; t \geq 0$.
(iii) If $E(0) > 0$, we first note that

$$2\int_0^t \int_{\Omega} uu_t\,dxd\tau = \|u\|^2 - \|u_0\|^2. \tag{17}$$

From Hölder's and Young's inequalities, we get

$$\|u\|^2 \leq \|u_0\|^2 + \int_0^t \|u\|^2 d\tau + \int_0^t \|u_\tau\|^2 d\tau. \tag{18}$$

By Hölder's and Young's inequalities, and (18), we get

$$a'(t) \leq a(t) + \|u_0\|^2 + \int_\Omega u_t^2\, dx + \int_0^t \|u_\tau\|^2\, d\tau. \tag{19}$$

Hence, by (12) and (19), we have

$$a''(t) - 4(\delta+1)a'(t) + 4(\delta+1)a(t) + K_1 \geq 0.$$

where

$$K_1 = 4(2\delta+1)E(0) + 4(\delta+1)\int_\Omega u_0^2\, dx - 4\delta \int_0^t \|u_\tau\|^2\, d\tau. \tag{20}$$

Let

$$b(t) = a(t) + \frac{K_1}{4(\delta+1)},\ t > 0.$$

Then $b(t)$ satisfies Lemma 4. Consequently, we get from (15) $a'(t) > \|u_0\|^2,\ t > 0$, where r_2 is given in Lemma 4.

Teorem 9. Assume $\frac{q-1}{4} \geq \delta \geq 0$ and one of the following statements are satisfied

(i) $E(0) < 0$ and $\int_\Omega u_0 u_1\, dx > 0$,

(ii) $E(0) = 0$ and $\int_\Omega u_0 u_1\, dx > 0$,

(iii) $0 < E(0) < \frac{\left(a'(t_0) - \|u_0\|^2\right)^2}{8[a(t_0) + (T_1 - t_0)\|u_0\|^2]}$ and (15) holds.

Then the solution u blow up in finite time T^* in the case of (10). In case (i),

$$T^* \leq t_0 - \frac{H(t_0)}{H'(t_0)}. \tag{21}$$

Furthermore, if $H(t_0) < min\left\{1, \sqrt{-\frac{a}{b}}\right\}$ we have

$$T^* \leq t_0 + \frac{1}{\sqrt{-b}} \ln \frac{\sqrt{-\frac{a}{b}}}{\sqrt{-\frac{a}{b}} - H(t_0)}, \tag{22}$$

where

$$a = \delta^2 H^{2+\frac{2}{\delta}}(t_0)\left[(a'(t_0) - \|u_0\|^2)^2 - 8E(0)H^{-\frac{1}{\delta}}(t_0)\right] > 0, \tag{23}$$
$$b = 8\delta^2 E(0). \tag{24}$$

In case (ii),

$$T^* \leq t_0 - \frac{H(t_0)}{H'(t_0)}. \tag{25}$$

In case (iii),

$$T^* \leq \frac{H(t_0)}{\sqrt{a}} \text{ or } T^* \leq t_0 + 2^{\frac{3\delta+1}{2\delta}}\left(\frac{a}{b}\right)^{2+\frac{1}{\delta}}\frac{\delta}{\sqrt{a}}\left\{1 - \left[1 + \left(\frac{a}{b}\right)^{2+\frac{1}{\delta}} H(t_0)\right]^{-\frac{1}{2\delta}}\right\}. \tag{26}$$

where a, b and $H(t)$ are defined in (23), (24) and (27), respectively.

Proof. Set

$$H(t) = [a(t) + (T_1 - t)\|u_0\|^2]^{-\delta}, \text{ for } t \in [0, T_1], \tag{27}$$

where $T_1 > 0$ is a certain constant which will be specified later. Then we get

$$\begin{aligned} H'(t) &= -\delta[a(t) + (T_1 - t)\|u_0\|^2]^{-\delta-1}[a'(t) - \|u_0\|^2] \\ &= -\delta H^{1+\frac{1}{\delta}}(t)[a'(t) - \|u_0\|^2], \end{aligned} \tag{28}$$

$$\begin{aligned} H''(t) = &-\delta H^{1+\frac{2}{\delta}}(t)a''(t)[a(t) + (T_1 - t)\|u_0\|^2] \\ &+\delta H^{1+\frac{2}{\delta}}(t)(1+\delta)[a'(t) - \|u_0\|^2]^2 \end{aligned} \tag{29}$$

and

$$H''(t) = -\delta H^{1+\frac{2}{\delta}}(t)V(t), \tag{30}$$

where

$$V(t) = a''(t)[a(t) + (T_1 - t)\|u_0\|^2] - (1+\delta)[a'(t) - \|u_0\|^2]^2. \tag{31}$$

For simplicity of calculation, we define

$$P_u = \int_\Omega u^2\, dx, \quad R_u = \int_\Omega u_t^2\, dx,$$

$$Q_u = \int_0^t \|u\|^2 d\tau, \quad S_u = \int_0^t \|u_\tau\|^2 d\tau.$$

By (13), (17) and Hölder's inequality, we have

$$a'(t) = 2\int_{\Omega} uu_t\,dx + \|u_0\|^2 + 2\int_0^t \int_{\Omega} uu_t dxdt$$
$$\leq \left(\sqrt{R_u P_u} + \sqrt{Q_u S_u}\right) + \|u_0\|^2. \quad (32)$$

If case (i) or (ii) holds, by (12) we have

$$a''(t) \geq (-4 - 8\delta)E(0) + 4(1+\delta)(R_u + S_u). \quad (33)$$

Thus, from (31)-(33) and (27), we obtain

$$V(t) \geq [(-4 - 8\delta)E(0) + 4(1+\delta)(R_u + S_u)]H^{-\frac{1}{\delta}}(t) - 4(1+\delta)\left(\sqrt{R_u P_u} + \sqrt{Q_u S_u}\right)^2.$$

From (11),

$$a(t) = \int_{\Omega} u^2\,dx + \int_0^t \int_{\Omega} u^2\,dx\,ds$$
$$= P_u + Q_u$$

and (27), we get

$$V(t) \geq (-4 - 8\delta)E(0)H^{-\frac{1}{\delta}}(t)$$
$$+4(1+\delta)[(R_u + S_u)(T_1 - t)\|u_0\|^2 + \theta(t)],$$

where

$$\theta(t) = (R_u + S_u)(P_u + Q_u) - \left(\sqrt{R_u P_u} + \sqrt{Q_u S_u}\right)^2$$

By the Schwarz inequality, and $\theta(t)$ being nonnegative, we have

$$V(t) \geq (-4 - 8\delta)E(0)H^{-\frac{1}{\delta}}(t), \quad t \geq t_0. \quad (34)$$

Therefore, by (30) and (34), we get

$$H''(t) \leq 4\delta(1 + 2\delta)E(0)H^{1+\frac{1}{\delta}}(t),\ t \geq t_0. \quad (35)$$

By Lemma 8, we know that $H'(t) < 0$ for $t \geq t_0$. Multiplying (35) by $H'(t)$ and integrating it from t_0 to t, we get

$$[H'(t)]^2 \geq a + bH^{2+\frac{1}{\delta}}(t)$$

for $t \geq t_0$, where a, b are defined in (23) and (24) respectively.
If case (iii) holds, similar to the steps of case (i), we get $a > 0$ if and only if

$$E(0) < \frac{(a'(t_0) - \|u_0\|^2)^2}{8[a(t_0) + (T_1 - t)\|u_0\|^2]}.$$

Then by Lemma 5, there exists a finite time T^* such that $\lim_{t \to T^{*-}} H(t) = 0$ and upper bound of T^* is estimated according to the sign of $E(0)$. This means that (10) holds.

4. Nonexistence of solutions with negative initial energy

In this section, we prove global nonexistence with negative initial energy.

Lemma 10. Suppose that $\psi(t)$ is a twice continuously differentiable function satisfying

$$\begin{cases} \psi''(t) + \psi'(t) \geq C_0 \psi^{1+\alpha}(t), \;\; t > 0, \;\; C_0 > 0, \;\; \alpha > 0, \\ \psi(0) > 0, \;\; \psi'(0) \geq 0 \end{cases} \tag{36}$$

where $C_0 > 0, \alpha > 0$ are constants. Then $\psi(t)$ blows up in finite time.

Proof. See [16].

Teorem 11. In addition to the conditions of Theorem 3, if

$$E(0) \leq 0 \text{ and } \int_\Omega u_0 \, u_1 \, dx \geq 0$$

then the corresponding solution blows up in finite time.

Proof. Multiplying Eq. (1) by u_t, and integration by parts, we have

$$\frac{d}{dt}\left[\frac{1}{2}\|u_t\|^2 + \frac{1}{2}\|\Delta u\|^2 + \frac{1}{2}\|\nabla u\|^2 - \frac{1}{q+1}\|u\|_{q+1}^{q+1}\right] = -\|u_t\|_2^2.$$

So the corresponding energy to problem (1) is defined as

$$E(t) = \frac{1}{2}\|u_t\|^2 + \frac{1}{2}(\|\Delta u\|^2 + \|\nabla u\|^2) - \frac{1}{q+1}\|u\|_{q+1}^{q+1}$$

and one can find that $E(t) \leq E(0)$ easily from

$$\int_0^t E'(t) = -\int_0^t \|u_\tau\| d\tau \leq 0.$$

Let

$$\psi(t) = \frac{1}{2}\int_\Omega \|u\|^2\, dx, \tag{37}$$

where u is a solution construct in theorem of Local existence. One can see that the derivative of $\psi(t)$ with respect to time

$$\psi'(t) = \int_\Omega uu_t\, dx \tag{38}$$

is well defined and Lipschitz continuous. Moreover, one can get by (37) and (38)

$$\begin{aligned}\psi''(t) &= \int_\Omega u_t^2\, dx + \int_\Omega |u|^{q-1}u^2\, dx - \int_\Omega u\Delta^2 u\, dx \\ &\quad + \int_\Omega u\Delta u\, dx - \int_\Omega uu_t\, dx \\ &= \|u_t\|^2 + \|u\|_{q+1}^{q+1} - \|\Delta u\|_2^2 - \|\nabla u\|_2^2 - \int_\Omega uu_t\, dx \\ &= \|u_t\|^2 + \|u\|_{q+1}^{q+1} - \|\Delta u\|_2^2 - \|\nabla u\|_2^2 - \psi'(t)\end{aligned}$$

and here we can write,

$$\psi''(t) + \psi'(t) = \|u_t\|^2 + \|u\|_{q+1}^{q+1} - \|\Delta u\|_2^2 - \|\nabla u\|_2^2.$$

If we substituting and adding $2E(t)$ to the right side of the equation, we get

$$\psi''(t) + \psi'(t) = 2\|u_t\|^2 - 2E(t) + \frac{q-1}{q+1}\|u\|_{q+1}^{q+1}.$$

Due to the

$\|u_t\|^2 \geq 0$ and $E(t) \leq 0$

conditions, we can write

$$\psi''(t) + \psi'(t) \geq \frac{q-1}{q+1}\|u\|_{q+1}^{q+1}. \tag{39}$$

Let's make an estimate for the term of $\|u\|_{q+1}^{q+1}$ in this expression. By Hölder's inequality,

$$\int_\Omega |u|^2\, dx \leq \left(\int_\Omega |u|^{q+1}\, dx\right)^{\frac{2}{q+1}} \left(\int_\Omega dx\right)^{\frac{q+1}{2}},$$

$$\|u\|_{q+1}^{q+1} \geq \left(\int_\Omega |u|^2\, dx\right)^{\frac{q+1}{2}} |\Omega|^{\frac{1-q}{2}}. \tag{40}$$

If the expression (40) is written at (39), thus

$$\psi''(t) + \psi'(t) \geq \frac{q-1}{q+1}|\Omega|^{\frac{1-q}{2}}\left(\int_\Omega |u|^2\,dx\right)^{\frac{q+1}{2}}$$
$$= 2^{\frac{q+1}{2}}\frac{q-1}{q+1}|\Omega|^{\frac{1-q}{2}}[\psi(t)]^{\frac{q+1}{2}},$$

$$\psi''(t) + \psi'(t) \geq C_0\psi^{1+\alpha}(t).$$

Then by Lemma 10 with $E(0) \leq 0$ and $\int_\Omega\ u_0\, u_1\, dx \geq 0$

$C_0 = 2^{\frac{q+1}{2}}\frac{q-1}{q+1}|\Omega|^{\frac{1-q}{2}}$ and $\alpha = \frac{q-1}{2}$,

we see that $\psi(t)$ blows up in finite time.

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